Name: **Version #1**

Instructor: Annette McP

Math 10120 Exam 3
Nov. 09, 2021.

- The Honor Code is in effect for this examination. All work is to be your own.
- Please turn off all cellphones and electronic devices.
- Calculators **are** allowed.
- The exam lasts for 1 hour and 15 minutes.
- Be sure that your name and your instructor's name are on the front page of your exam.
- Be sure that you have all 13 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(•)	(b)	(c)	(d)	(e)
2.	(•)	(b)	(c)	(d)	(e)
3.	(•)	(b)	(c)	(d)	(e)
4.	(•)	(b)	(c)	(d)	(e)
5.	(•)	(b)	(c)	(d)	(e)
6.	(•)	(b)	(c)	(d)	(e)
7.	(•)	(b)	(c)	(d)	(e)
8.	(•)	(b)	(c)	(d)	(e)
9.	(•)	(b)	(c)	(d)	(e)
10.	(•)	(b)	(c)	(d)	(e)

Please do NOT write in this box.

Multiple Choice	______
11.	______
12.	______
13.	______
14.	______
15.	______
Total	______

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3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
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15.	______
Total	______

Multiple Choice

1.(5pts) The following frequency table shows the number of pets owned by 30 students at Notre Dame. Find the mean, $\bar{x}$, for the set of data.

$x = \#Pets$	Frequency	$x \cdot$ Freq.
0	8	0
1	5	5
2	7	14
3	5	15
4	1	4
5	3	15
10	1	10
		63

$\rightarrow \bar{x} = \frac{63}{n} = \frac{63}{30} = \frac{21}{10} = 2.1$

(a) 2.1 (b) 3.1 (c) 1.9 (d) 4.2 (e) 0.833

2.(5pts) The Temperature readings at Noon in the town of East Bend for a random sample of 20 days are given in the following table:

$n = 20$ $\bar{x} = 54$ GIVEN

Temerature	Frequency	$x - \bar{x}$	$(x-\bar{x})^2$	$(x-\bar{x})^2$ freq
50	2	-4	16	32
52	6	-2	4	24
55	9	+1	1	9
56	1	2	4	4
57	1	3	9	9
60	1	6	36	36
				114

The average temperature in the sample is $\bar{x} = 54$ (there is no need to check this). What is the sample standard deviation ?

$s^2 = \frac{114}{n-1} = \frac{114}{19}$

(a) $s = \sqrt{\frac{114}{19}}$ (b) $s = \sqrt{\frac{70}{19}}$ (c) $s = \sqrt{\frac{124}{19}}$ (d) $s = \sqrt{\frac{70}{20}}$ (e) $s = \sqrt{\frac{75}{20}}$

$s = \sqrt{s^2}$

$= \sqrt{\frac{114}{19}}$

3.(5pts) The probability distribution of the random variable X is shown in the accompanying table.

x	0	5	10	15	20
$\Pr(X=x)$	0.21	0.16	0.18	0.21	0.24

$P(X>5)$
$= .18 + .21 + .24$
$= .63$

Which statement (a)-(e) is correct?

(a) $P(X>5)=0.63$ (b) $P(X>5)=0.79$ (c) $P(X>5)=0.21$

(d) $P(X>5)=0.37$ (e) $P(X>5)=0.16$

4.(5pts) A box contains 3 red marbles and 2 blue marbles. A sample of two marbles is drawn at random from the box. Let X denote the number of blue marbles in the sample. Which of the following gives the probability distribution for the random variable X?

(a)

X	P(X)
0	3/10
1	6/10
2	1/10

(b)

X	P(X)
0	17/32
1	5/32
2	10/32

(c)

X	P(X)
0	1/10
1	6/10
2	3/10

(d)

X	P(X)
0	1/5
1	2/5
2	2/5

(e)

X	P(X)
0	10/32
1	5/32
2	17/32

Tot. = 5

3R	2B

Sample Size = 2

X = # B in Sample

Possible values of X are 0, 1, 2.

X	P(X)
0	3/10
1	6/10
2	1/10

$P(x=1) = P(1R\,1B)$

$= \dfrac{C(3,1)\,C(2,1)}{C(5,2)} = \dfrac{3\cdot 2}{10} = \dfrac{6}{10}$

$P(x=2) = P(2B) = \dfrac{C(2,2)}{C(5,2)}$
$= \dfrac{1}{10}$

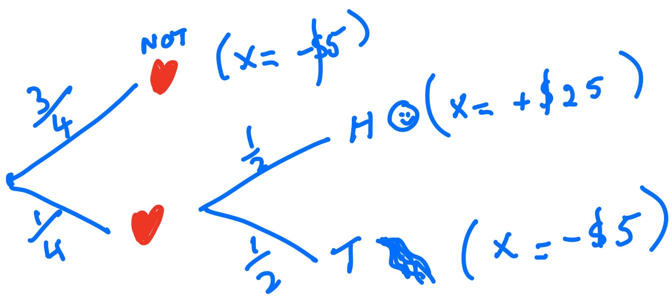
$P(x=0) = P(2R)$
$= C(3,2)/C(5,2) = \dfrac{3}{10}$

5.(5pts) A carnival game has the following rules:

- The player pays \$5 to play the game.
- The player then draws a card from a well shuffled standard deck of 52 cards.
- If the card drawn is not a heart, the game is over and the player loses.
- If the card drawn is a heart, the player flips a coin.
- If the player gets a tail on the coin, the game is over and the player loses.
- If the player gets a head on the coin, the player wins and receives \$30 from the game attendant.

Let X be the (net) earnings for the player in this game. What is the expected value of X? (A tree diagram might help).

~~(a)~~ - \$1.25 (b) -\$0.625 (c) \$3.75 (d) \$15 (e) -\$1.875



X	$P(x)$	$xP(x)$
-5	$\frac{3}{4}+\frac{1}{4}\cdot\frac{1}{2}=\frac{7}{8}$	$-\frac{35}{8}$
25	$\frac{1}{4}\cdot\frac{1}{2}=\frac{1}{8}$	$+\frac{25}{8}$
		$=-10/8$
		$\boxed{=-1.25}$

6.(5pts) If Z is a standard normal random variable, what is $P(-0.25 \le Z \le 0.25)$.

Note You will find tables for the standard normal distribution at the end of the exam.

~~(a)~~ 0.1974 (b) 0.5987 (c) 0.4013 (d) 0.2417 (e) 0.9332

$$P(-0.25 \le Z \le 0.25)$$
$$= A(0.25) - A(-0.25)$$
$$= .5987 - .4013 \text{ (FROM TABLES)}$$
$$= .1974$$

7.(5pts) The height (at the shoulder) of adult snopalopagus' is normally distributed with mean $\mu = 9$ ft. and standard deviation $\sigma = 3$ ft. If I choose an adult snopalopagus at random from the population, what is the probability that it will have a shoulder height greater than 15.3 feet?

(a) 0.0179 (b) 0.0359 (c) 0.9821 (d) 0.4591 (e) 0.1358

$\mu = 9, \quad \sigma = 3, \quad X = \text{height}$

$$P(X > 15.3) = P\left(Z > \frac{15.3 - 9}{3}\right)$$

$$= P(Z > 2.1)$$

$$= 1 - A(2.1) = 1 - .9821$$

$$= \boxed{.0179}$$

8.(5pts) Which of the following pairs of values (x, y) is in the feasible set of the following system of inequalities?

$$\begin{cases} x \geq 1 \\ y \geq 0 \\ x + y \geq 5 \\ x + 2y \geq 6 \end{cases}$$

	(2,3)	(2,−2)	(1,3)	(4,0)	(0,100)
$x \geq 1$	✓	✓	✓	✓	×
$y \geq 0$	✓	×	✓	✓	
$x + y \geq 5$	✓		×	×	
$x + 2y \geq 6$	✓				

(a) $(2, 3)$ (b) $(2, -2)$ (c) $(1, 3)$ (d) $(4, 0)$ (e) $(0, 100)$

X is binomial, $n=10$, $p=\frac{1}{6}$, $X = \#$ "6"'s

9.(5pts) A fair six sided die is tossed 10 times. What is the probability of getting <u>at most</u> eight sixes?

(a) 1- $\left[C(10,9)\left(\frac{1}{6}\right)^9\left(\frac{5}{6}\right)^1 + C(10,10)\left(\frac{1}{6}\right)^{10}\left(\frac{5}{6}\right)^0 \right]$

(b) $C(10,8)\left(\frac{1}{6}\right)^8\left(\frac{5}{6}\right)^2$

(c) $C(10,8)\left(\frac{1}{6}\right)^8\left(\frac{5}{6}\right)^2 + C(10,9)\left(\frac{1}{6}\right)^9\left(\frac{5}{6}\right)^1 + C(10,10)\left(\frac{1}{6}\right)^{10}\left(\frac{5}{6}\right)^0$

(d) $C(10,9)\left(\frac{1}{6}\right)^9\left(\frac{5}{6}\right)^1 + C(10,10)\left(\frac{1}{6}\right)^{10}\left(\frac{5}{6}\right)^0$

(e) $\left(\frac{1}{6}\right)^8$

$P(X \leq 8)$

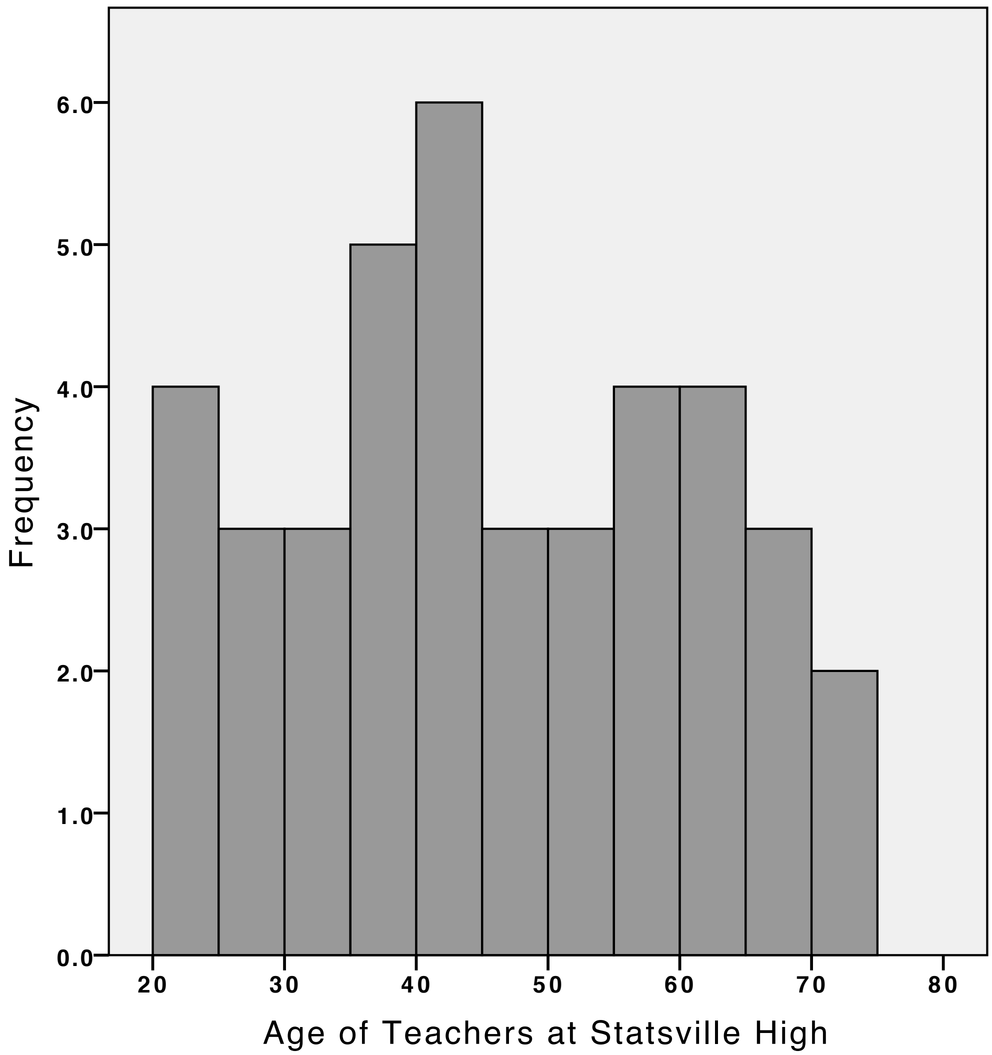
$= 1 - P(X > 8)$

$= 1 - [P(X=9) + P(X=10)]$

$= 1 - \left[C(10,9)\left(\frac{1}{6}\right)^9\left(\frac{5}{6}\right)^1 + C(10,10)\left(\frac{1}{6}\right)^{10}\left(\frac{5}{6}\right)^0 \right]$

10.(5pts) The histogram shown below, gives the frequency of age groups for all teachers at Statsville High School. There 40 teachers at the high school. (We assume that the histogram follows the convention that each category is an interval that contains it's left end point but not it's right end point; for example the age category on the extreme left is $[20, 25)$.)

$n = 40$



Which of the following statements can deduced from the information given in the histogram?

15 Teachers < 40 and 25 are ≥ 40 ($n = 40$ teachers)

~~(a)~~ At least half of the teachers are 40 or older.

(b) At least one teacher is 25 years old. cannot say That; 3 in cat $[25, 30)$

(c) More than three quarters of the teachers are younger than 35. No less than half of them are younger than 40

(d) Six of the teachers are exactly 40 years old

6 are in the cat $[40, 45)$ but have no idea of their exact age

(e) At least one teacher is 75 years old.

No actually we know this is untrue since there are none in the cat $[75, 80)$

For Questions 11-14, you may express your answers using the notation for permutations, combinations, powers and factorials, where appropriate

11.(12pts) In the upcoming presidential elections in Mathland (which has a voting population of about 100 thousand), 45% are currently planning to vote for Melinda McNulty and 55% are planning to vote for somebody else. In a polling experiment, Kyle chooses a member of the voting population at random and asks if they are planning to vote for Melinda in the next election. He repeats this experiment 10 times.

$X = \#$ in sample who will vote for Melinda

(a) What is the probability that exactly 5 of those interviewed are planning to vote for Melinda McNulty?

X is a binomial R.V., $n = 10$, $p = .45$

$$P(X=5) = C(10,5)(.45)^5(.55)^5$$

(b) What is the probability that at least 5 of those interviewed are planning to vote for Melinda McNulty?

$$P(X \geq 5) = P(X=6) + P(X=7) + \cdots + P(X=10)$$

$$= C(10,5)(.45)^5(.55)^5 + C(10,6)(.45)^6(.55)^4 + C(10,7)(.45)^7(.55)^3 + C(10,8)(.45)^8(.55)^2$$
$$+ C(10,9)(.45)^9(.55)^1 + C(10,10)(.45)^{10}(.55)^0$$

(c) Kyle, who is in charge of Melinda's election campaign, wants to devote some of the campaign fund to advertising. He will distribute the money between advertisements on TV and social media platforms. It is estimated that each dollar spent on TV ads will attract 0.5 new voters and each dollar spent on social media ads will attract 0.7 new voters. Let \$x denote the number of dollars that Kyle devotes to TV ads and let \$y denote the number of dollars that Kyle will devote to social media ads. Kyle needs to attract at least 10,000 new voters with the advertising campaign. Express this constraint as an inequality in the variables x and y.

$$0.5x + 0.7y \geq 10,000$$

12.(12pts) The MCAT exam has 4 sections. Test scores on each section are normally distributed. The table below shows the mean and standard deviations for the four sections of the exam from 2015 to 2017.
(A table of values for the normal distribution is given at the end of the exam.)

		Mean (μ)	Standard Deviation (σ)
Y	Chemical and Physical Foundations of Biological Systems	125.1	3.0
X	Critical Analysis and Reasoning Skills	124.8	2.9
	Biological and Biochemical Foundations of Living Systems	125.3	3.0
W	Psychological, Social, and Biological Foundations of Behavior	125.4	3.1

X = Score on C. An. + Reas.

(a) What percentage of students who took the exam between 2015 and 2017 got a score between 119 and 129.15 on the Critical Analysis and Reasoning Skills section?

$\mu = 124.8 \quad \sigma = 2.9$

$$P(119 \le X \le 129.15) = P\left(\frac{119-124.8}{2.9} \le Z \le \frac{129.15-124.8}{2.9}\right)$$

$$= P(-2 \le Z \le 1.5)$$

$$= A(1.5) - A(-2) = .9332 - .0228$$

$$= \boxed{.9104}$$

$\boxed{91.04\%}$

(b) Mary got a score of 130.05 on the Chemical and Physical Foundations of Biological Systems section in 2016. What percentage of students who took the exam between 2015 and 2017 got a score less than or equal to Mary's on that section?

Y = Score
$\mu = 125.1$
$\sigma = 3.0$

$$P(Y \le 130.05) = P\left(Z \le \frac{130.05 - 125.1}{3.0}\right)$$

$$= P(Z \le 1.65) = A(1.65)$$

$$= \boxed{.9505}$$

$\boxed{95.05\%}$

(c) Mary also got a score of 130.05 on the Psychological, Social, and Biological Foundations of Behavior section. In which of the two sections; Chemical and Physical Foundations of Biological Systems, or Psychological, Social, and Biological Foundations of Behavior sections, did Mary have a higher standardized score? (Please justify your answer.)

W
$\mu = 125.4$
$\sigma = 3.1$

Mary's z score on Psyc., Soc. + Bio Fnd.s

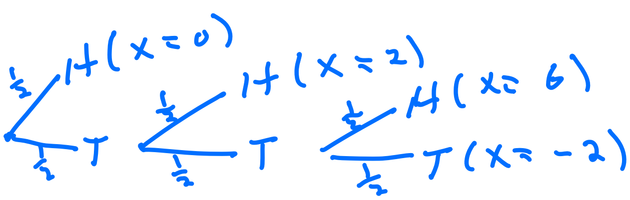
$$= \frac{130.05 - 125.4}{3.1} = 1.5$$

Mary had a higher z score of 1.65 on Chem + Phy Fnd.s of Bio. Syts.

Initials: ____________

13.(12pts) At a fairground game stand, you pay \$2 to play.
You then flip a fair coin until you get a head
or until you have flipped the coin 3 times (whichever comes first).
The game attendant gives you \$2 if you get a head on the first flip,
\$4 if you get a head on the second flip,
and \$8 if you get a head on the third flip.
If you do not get a head, the game attendant gives you nothing.
Let X denote the (net) earnings for the player for one play of this game.

(a) Find the probability distribution for the random variable X.



X	Pr(X)	X P(X)
-2	$\frac{1}{8}$	$-2/8$
0	$\frac{1}{2}$	0
2	$\frac{1}{4}$	$\frac{2}{4} = \frac{4}{8}$
6	$\frac{1}{8}$	$\frac{6}{8}$

$E(X) = +\frac{8}{8} = 1$

(b) What is $E(X)$ (the expected value of X) for the probability distribution you found in Part (a)?

$E(X) = 1$

(c) If you played this game 50 times, how much would you expect to win?

$50 \times \$1 = \$50$

14.(12pts) The following data set shows the times of the top 50 competitors in the Holy Half Marathon for 2019 (the times are rounded to the nearest minute).

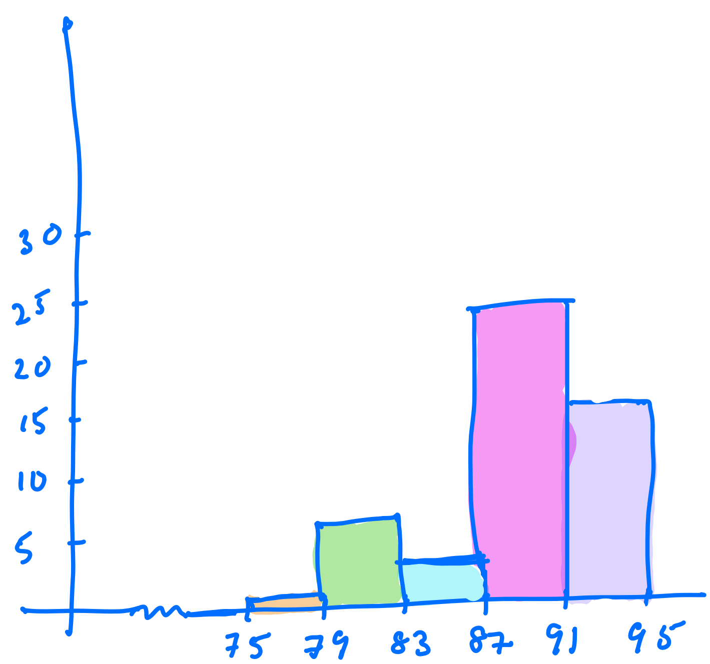
75	79	79	80	80	81	81	83	83	86
87	87	87	88	88	88	88	88	88	89
89	89	89	89	89	90	90	90	90	90
90	90	90	90	91	91	91	91	92	92
92	92	92	92	92	92	92	93	93	93

(a) What is the median of this data set?

$$M = \frac{89+90}{2} = 89.5$$

(b) Create a histogram for this data set using **5 categories of equal length** (intervals of the form [,) including the left end point in each category). Fill in the intervals and frequencies in the table on the left and draw your histogram in the box on the right.

(Category)	(Frequency)
[75, 79)	1
[79, 83)	6
[83, 87)	3
[87, 91)	24
[91, 95)	16
Total	



Cat Length

$$\frac{93-75}{5} = 3.6 \xrightarrow{\text{inc to}} 4$$

If start @ 75 ... end @ 95

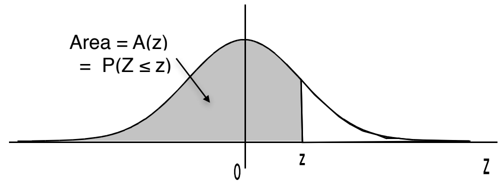
o.k ✓

15.(2pts) You will be awarded these two points if you write your name in CAPITALS and you mark your answers on the front page with an X (not an O) . You may also use this page for

ROUGH WORK

Initials: ____________

Areas under the Standard Normal Curve



z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$	z	$A(z)$
−3.50	.0002	−2.00	.0228	−.50	.3085	1.00	.8413	2.50	.9938
−3.45	.0003	−1.95	.0256	−.45	.3264	1.05	.8531	2.55	.9946
−3.40	.0003	−1.90	.0287	−.40	.3446	1.10	.8643	2.60	.9953
−3.35	.0004	−1.85	.0322	−.35	.3632	1.15	.8749	2.65	.9960
−3.30	.0005	−1.80	.0359	−.30	.3821	1.20	.8849	2.70	.9965
−3.25	.0006	−1.75	.0401	−.25	.4013	1.25	.8944	2.75	.9970
−3.20	.0007	−1.70	.0446	−.20	.4207	1.30	.9032	2.80	.9974
−3.15	.0008	−1.65	.0495	−.15	.4404	1.35	.9115	2.85	.9978
−3.10	.0010	−1.60	.0548	−.10	.4602	1.40	.9192	2.90	.9981
−3.05	.0011	−1.55	.0606	−.05	.4801	1.45	.9265	2.95	.9984
−3.00	.0013	−1.50	.0668	.00	.5000	1.50	.9332	3.00	.9987
−2.95	.0016	−1.45	.0735	.05	.5199	1.55	.9394	3.05	.9989
−2.90	.0019	−1.40	.0808	.10	.5398	1.60	.9452	3.10	.9990
−2.85	.0022	−1.35	.0885	.15	.5596	1.65	.9505	3.15	.9992
−2.80	.0026	−1.30	.0968	.20	.5793	1.70	.9554	3.20	.9993
−2.75	.0030	−1.25	.1056	.25	.5987	1.75	.9599	3.25	.9994
−2.70	.0035	−1.20	.1151	.30	.6179	1.80	.9641	3.30	.9995
−2.65	.0040	−1.15	.1251	.35	.6368	1.85	.9678	3.35	.9996
−2.60	.0047	−1.10	.1357	.40	.6554	1.90	.9713	3.40	.9997
−2.55	.0054	−1.05	.1469	.45	.6736	1.95	.9744	3.45	.9997
−2.50	.0062	−1.00	.1587	.50	.6915	2.00	.9772	3.50	.9998
−2.45	.0071	−.95	.1711	.55	.7088	2.05	.9798		
−2.40	.0082	−.90	.1841	.60	.7257	2.10	.9821		
−2.35	.0094	−.85	.1977	.65	.7422	2.15	.9842		
−2.30	.0107	−.80	.2119	.70	.7580	2.20	.9861		
−2.25	.0122	−.75	.2266	.75	.7734	2.25	.9878		
−2.20	.0139	−.70	.2420	.80	.7881	2.30	.9893		
−2.15	.0158	−.65	.2578	.85	.8023	2.35	.9906		
−2.10	.0179	−.60	.2743	.90	.8159	2.40	.9918		
−2.05	.0202	−.55	.2912	.95	.8289	2.45	.9929		